

# 4.8 Trig Applications

**Simple Harmonic Motion:** A point that moves on a coordinate line is said to be in simple harmonic motion if its distance  $d$  from the origin at time  $t$  is given by either the sine or cosine function.

- 1) A buoy oscillates in simple harmonic motion as waves go past. It is noted that the buoy moves a total of 3.5 feet from its low point to its high point, and that it returns to its high point every 10 seconds. Write an equation that describes the motion of the buoy if its high point is at  $t=0$ .



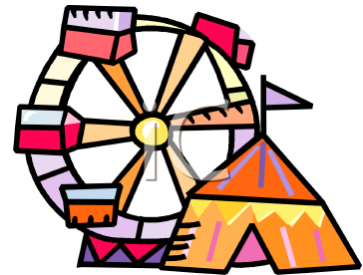
- 2) A ball that is bobbing up and down on the end of a spring has a maximum displacement of 3 inches. Its motion is modeled by  $y = \frac{1}{4} \cos 16t$  ( $t > 0$ ), where  $y$  is measured in feet and  $t$  is time in seconds.
- What is the period?
  - Determine the first time the weight passes the point of equilibrium ( $y=0$ ).
- 3) Suppose one day all 300 million people in the US climb up on tables. At time  $t=0$ , we all jump off. The resulting shock as we hit the earth's surface will start vibrating the entire Earth in such a way that its surface first moves **down** from its normal position and then moves up an equal distance **above** its normal position. The displacement ( $y$ ) of the surface is a sinusoidal function of time with a period of 54 minutes. The amplitude is 50 meters. Write an equation to represent this situation.
- 4) The function given by  $P = 100 - 20 \cos \frac{5\pi}{3} t$  approximates the blood pressure  $P$  (in millimeters) of mercury at time  $t$  (in seconds) for a person at rest.
- Find the period of the function.
  - Find the number of heartbeats per minute.

- 5) The voltage  $E$  in an electrical circuit is modeled by  $E=5\cos 120\pi t$ , where  $t$  is measured in seconds.
- Find the amplitude and period.
  - How many cycles are completed in 1 second? (The number of cycles completed in 1 second is the **frequency** of the function)
  - Find  $E$  when  $t=0, .03, .06, .09, .12$

- 6) A Ferris wheel is built such that the height  $h$  (in feet) above ground of a seat on the wheel at time  $t$  (in seconds) can be modeled by  $h(t) = 53 + 50\sin(\frac{\pi}{10}t - \frac{\pi}{2})$
- Find the period of the model. What does the period tell you in terms of the ride?

b) Find the amplitude of the model. What does the amplitude tell you about the ride?

c) How long would it take to complete 5 revolutions around?



- 7) A satellite is deployed from a space shuttle into an orbit which goes alternately north and south of the equator. Its distance from the equator over time can be approximated by a sine wave. It reaches 4500 km, its farthest point north of the equator, 15 minutes after the launch. Half an orbit later it is 4500 km south of the equator, its farthest point south. One complete orbit takes 1 hour.
- Find an equation of a sinusoidal function that models the distance of the satellite from the equator.
  - How far away from the equator is the satellite 1.25 hours after launch?

## 4.8 Trig Applications Homework

- 1) For a person at rest, the velocity  $v$  (in liters per second) of air flow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is given by  $v = 0.85\sin\frac{\pi}{3}t$  (where  $t$  is the time in seconds).
  - a) Find the time for one full respiratory cycle.
  - b) Find the number of cycles per minute.
  
- 2) The voltage  $E$  in an electrical circuit is modeled by  $E=3.8\cos 40\pi t$ , where  $t$  is the time measured in seconds.
  - a) Find the amplitude and period.
  - b) Find the frequency.
  - c) Find  $E$  when  $t=.02, .04, .08, .12, .14$
  
- 3) A signal buoy moves a distance of 4.5 feet from its highest point to its lowest point. It moves from its highest point to its lowest point and back to its highest point every 14 seconds. Write an equation that describes the motion of the buoy if its high point is at  $t=0$ .
  
- 4) As you ride a Ferris wheel, the height that you are above ground varies periodically. Consider the point at the center of the wheel to be the equilibrium point. Suppose the diameter of the Ferris wheel is 43 feet and travels at a rate of 3 revolutions per minute. At the highest point, a seat on the Ferris wheel is 46 feet above the ground.
  - a) What is the lowest height of the seat?
  - b) What is the period of the function?
  - c) Write a sine equation to model the height of the seat.
  - d) According to the model, what is the height of the seat after 10 seconds?
  
- 5) Your fishing bobber oscillates in simple harmonic motion from the waves in the lake. Your bobber moves a total of 1.5 inches from its high point to its low point and returns to its high point every 3 seconds. Write an equation modeling the motion of the bobber if it is at its high point at time  $t=0$ .